MORE DEPTH ON THE MATHEMATICS:

The purpose of this part of the website is to provide greater depth of the certain mathematical developments of some chapters of the book. Please contact the author at faithequation@gmail.com if you have further inquiries.

Chapter 4: *The Issue of Independent Events*. An opposition might arise in the mind of the reader with a background in the theory of probability and statistics. It concerns the assumption of independent events when obtaining the overall probability of the 9 prophecies being fulfilled by multiplying the 9 individual prophecies. One might reasonably assume that if one prophecy has been fulfilled, it might increase the likelihood that a second prophecy will be fulfilled, and if two prophecies have been fulfilled, it is even more likely that a third prophecy will be fulfilled. In general, does the fact that some prophecies are known to have been fulfilled increase the probability that subsequent prophecies will be fulfilled? We consider this possibility using an argument based on conditional probability.

1) Given that one prophecy has been fulfilled, the conditional probability that another prophecy will be fulfilled *increases* by an arbitrary percentage $q_1(100\%)$, $0 \le q_1 \le 1$, *relative* to the unconditional probability of fulfilling this prophecy. Thus, if one prophecy has occurred, and the unconditional probability of the second prophecy is p_2, the conditional probability that the 2nd prophecy will be fulfilled given that the first prophecy *has been* fulfilled is $p_2(1 + q_1)$.

Note: Since q_1 = the increase in conditional probability of the 2nd prophecy occurring (given that the 1st prophecy has occurred) *relative* to the unconditional probability of fulfilling this 2nd prophecy (i.e., p_2), the *actual increase* in the conditional probability (over the unconditional probability) is p_2q_1. Thus the conditional probability that the 2nd prophecy will occur given that the first has occurred is

$$p_2 + p_2q_1 = p_2(1 + q_1).$$

Note that

$$\frac{\text{Cond. Pr. of } 2^{nd} \text{ Prophecy Given } 1^{st} \text{ Fulfilled)} - \text{Uncond. Pr. of } 2^{nd} \text{ Prophecy}}{\text{Uncond. Pr. of } 2^{nd} \text{ Prophecy}} = \frac{p_2(1+q_1) - p_2}{p_2} = q_1$$

Thus, the joint probability that the first two prophecies are fulfilled is

$$p_1p_2(1 + q_1)$$

2. Given that two prophecies have been fulfilled, the conditional probability that a third prophecy will be fulfilled increases by q_2(100%), $0 \le q_2 \le 1$, *relative* to the conditional probability of this prophecy being fulfilled given that only one prophecy has been fulfilled. Thus, if p_3 denotes the unconditional probability that the 3rd prophecy is fulfilled, and $p_3(1 + q_1)$ denotes the conditional probability that the 3rd prophecy is fulfilled given that the 1st prophecy has been fulfilled, then $p_3(1 + q_1)(1 + q_2)$ is the conditional probability that the 3rd prophecy will be fulfilled given that both the 1st and 2nd prophecies have been fulfilled. Note that

$$\frac{\text{Cond. Pr. of } 3^{rd} \text{ Prophecy Given } 1^{st} \;\&\; 2^{nd} \text{ Fulfilled)} - \text{Cond. Pr. of } 3^{rd} \text{ Prophecy Given } 1^{st} \text{ Fulfilled}}{\text{Cond. Pr. of } 3^{rd} \text{ Prophecy Given } 1^{st} \text{ Fulfilled}}$$

$$= \frac{p_3(1+q_1)(1+q_2) - p_3(1+q_1)}{p_3(1+q_1)} = q_2 \; .$$

Thus, the joint probability that the first three prophecies are fulfilled is

$$p_1 p_2 (1 + q_1) p_3 (1 + q_1)(1 + q_2) = p_1 p_2 \, p_3 \, (1 + q_1)^2 (1 + q_2).$$

3. Now given that three prophecies have been fulfilled, the conditional probability that a 4th prophecy will be fulfilled increases by q_3(100%), $0 \le q_3 \le 1$, *relative* to the conditional probability of this prophecy being fulfilled given that only two prophecies have been fulfilled. Thus, if p_4 denotes the unconditional probability that the 4th prophecy is fulfilled, and $p_4(1 + q_1)$ denotes the conditional probability that the 4th prophecy is fulfilled given that the 1st prophecy has been fulfilled, and the conditional probability that the 4th prophecy is fulfilled given that the 1st and 2nd prophecies have been fulfilled is , $p_3(1 + q_1)(1 + q_2)$ then $p_4(1 + q_1)(1 + q_2)(1 + q_3)$ is the conditional probability that the 4th prophecy will be fulfilled given that the 1st, 2nd and prophecies have been fulfilled. Note that

$$\frac{\text{Cond. Pr. of } 4^{rth} \text{ Prophecy Given } 1^{st} \; 2^{nd} \;\&\; 3^{rd} \text{ Fulfilled)} - \text{Cond. Pr. of } 4^{rth} \text{ Prophecy Given } 1^{st} \;\&\; 2^{nd} \text{ Fulfilled}}{\text{Cond. Pr. of } 3^{rd} \text{ Prophecy Given } 1^{st} \;\&\; 2^{nd} \text{ Fulfilled}}$$

$$= \frac{p_3(1+q_1)(1+q_2)(1+q_3) - p_3(1+q_1)(1+q_2)}{p_3(1+q_1)(1+q_2)} = q_3 \; .$$

Therefore, the joint probability that the first 4 prophecies are fulfilled is

$$p_1p_2(1 + q_1)p_3(1 + q_1)(1 + q_2)p_4(1 + q_1)(1 + q_2)(1 + q_3)$$
$$= p_1p_2p_3p_4\,(1 + q_1)^3(1 + q_2)^2(1 + q_3).$$

Extending this argument we see that the conditional probability of the n^th^ prophecy being fulfilled given that the previous $n-1$ prophecies have been fulfilled is

$$p_n(1 + q_1)\,(1 + q_2)\ldots\ldots\ldots\,(1 + q_{n-1}),$$

where p_n denotes the unconditional probability of the n^{th} prophecy being fulfilled.

Thus, by the Multiplication Rule, the probability of n specific prophecies being fulfilled is

$$p_1\,p_2(1 + q_1)\;\;p_3(1 + q_1)^2\,(1 + q_2)\,\ldots p_n(1 + q_1)\,(1 + q_2)\ldots(1 + q_{n-1})$$

$$= p_1\,p_2\,\ldots\ldots\,p_n\,(1 + q_1)^{n-1}(1 + q_2)^{n-2}\,\ldots\ldots\ldots\,(1 + q_{n-1}).$$

Remark: Technically, since probabilities cannot exceed one, we should write the above conditional probability as

$$\text{Min}\{1,\, p_n(1 + q_1)\,(1 + q_2)\ldots\ldots\ldots\,(1 + q_{n-1})\}.$$

However, since typically p_n is quite small, we can reasonably assume that $p_n(1 + q_1)\,(1 + q_2)\ldots\ldots\ldots\,(1 + q_{n-1}) < 1$.

Now let

$$Q = \text{Max}\{q_1,\, q_2,\, \ldots\ldots\ldots,\, q_{n-1}\}.$$

Remark: Presumably as more and more probabilities are fulfilled, the conditional probability of the next one being fulfilled increases. Thus, it would appear that $Q = q_{n-1}$, but from a purely mathematical point of view, this need not necessarily be the case.

Now

$$p_1\,p_2\,\ldots\ldots\,p_n\,(1 + q_1)^{n-1}(1 + q_2)^{n-2}\,\ldots\ldots\ldots\,(1 + q_{n-1})$$
$$\leq\; p_1\,p_2\,\ldots\ldots\,p_n\,(1 + Q)(1 + Q)^2\,\ldots\ldots\ldots\,(1 + Q)^{n-1}$$
$$= p_1\,p_2\,\ldots\ldots\,p_n\,(1 + Q)^{1 + 2 + \ldots\ldots + n-1} = p_1\,p_2\,\ldots\ldots\,p_n\,(1 + Q)^{n(n-1)/2}.$$

In the case of 9 prophecies, $n = 9$, and the probability that all 9 are fulfilled is

$$p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6 \cdot p_7 \cdot p_8 \cdot p_9 (1 + Q)^{36} .$$

Now, if we were to assume that Q = 0.10, which in this context is relatively high, the conditional argument increases the probability that 9 prophecies are fulfilled by

$$(1.1)^{36} = 30.913 < 100 = 10^2 .$$

But, as already computed in Chapter 4, $p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6 \cdot p_7 \cdot p_8 \cdot p_9 = 10^{-76}$, and $100 \times 10^{-76} = 10^{-74}$ so the probability of these prophecies being fulfilled is still less than 10^{-74} (i.e., negligible). We reach the conclusion that any increase in the overall probability of these 9 prophecies being fulfilled is negligible, and that the assertion of independent events allowing multiplication of probabilities is valid.

<u>Remark</u>: Even if we assume Q = 0.2,

$$(1.2)^{36} = 708.8 < 1000 = 10^3 .$$

But since

$$p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6 \cdot p_7 \cdot p_8 \cdot p_9 = 10^{-76} ,$$

$$p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6 \cdot p_7 \cdot p_8 \cdot p_9 (1 + Q)^{36} \quad 10^{-76} \times 10^3 = 10^{-73},$$

which is still negligible.

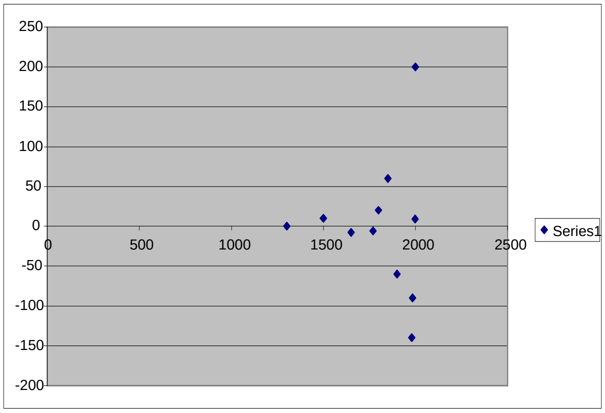
If the reader chooses to reject this reasoning, he/she should cling to the idea that the Bible contains hundreds if not thousands of prophecies which have come true, with none failing. That is staggering evidence that we should pay expect the other prophecies to come true, and that the Bible is a reliable document.

<u>Chapter 5:</u> *The Mathematics of the Curve Fitting - Advanced Approach*

For more mathematical depth about how the models in Chapter 5 were formed, let's retrace the curve fitting from a more advanced statistical approach, using calculus and statistical theory. Look again at the data in Table 5.1. Suppose we try to fit the data to a polynomial predicting function. To decide the appropriateness of this decision, consider the error function defined by

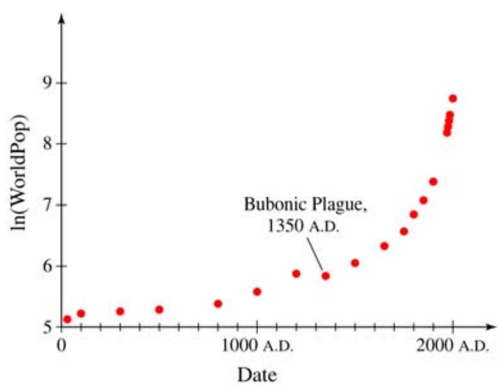
$$\text{ErrorFcn} = \text{Actual Data} - \text{Predicted Values}$$

We do a scatterplot of the ErrorFcn versus time, as follows.



Note that the function values tend to spread out as time increases. This indicates that by taking the natural logarithm of the data, we have can use a polynomial as a predicted function. When we take the natural log, the errors have to be constant, or within a bound. Taking the ln is a reasonable choice for an error stabilizing transformation, since growth tends to be exponential.

We take the ln of the data and form a scatterplot as follows.



We notice a "bump" in the data around 1350 AD where the population decreases. To better determine this "bump" we do a transformation of the data, considering the annualized growth rate:

$$\text{Annual Growth Rate} = \left(\frac{\text{Current Population}}{\text{Previous Population}}\right) \wedge \left(\frac{1}{\text{Current Year - Previous Year}}\right)$$

We do this annualizing because the distance between data points is not equal. To get the distances between data more equal we make this transformation. Then the bump point is vivid between 1200 and 1350. That "bump" point occurs around the time of the Bubonic Plague.

BUBONIC PLAGUE: The Black Death, or Bubonic Plague swept Europe and parts of Asia in the 14th century, killing as much as 75% of the population of Europe and parts of Asia in less than 20 years.

Because of these analyses we decide to restrict our data to 1350 AD and beyond.

To discover the form of the curve, note the following table regarding polynomial functions for curve fitting.

Degree of Polynomial	Number of Data Points Needed
2	3
3	4
n	n+1

Thinking of growth rate as the derivative of a polynomial, if we have $n+1$ data points, then a polynomial of degree n gives a perfect fit. Suppose we have 100 data points, then we know a polynomial of degree 99 will fit growth perfectly. But a statistician-mathematician would look for a more "elegant" fit by finding a polynomial of smaller degree if possible, especially in order to compute function values. To make this analysis, we consider the increasing pattern of r^2, where

$$r^2 = \frac{\sum_{t=1350}^{2000} (\hat{y}_t - \bar{y})^2}{\sum_{t=1350}^{2000} (y_t - \bar{y})^2}, \quad \hat{y}_t = e^{\text{Quartic}(t)}$$

From statistical theory, it is know that

$\lim r^2 = 1$

(As the degree in the regression poly $\Rightarrow n+1$).

Here, the cubic fit has $r^2 = 0.973$, but quadratic fit has $r^2 = 0.943$. Since there is only a small percent change in the difference $1 - r^2$ from a cubic to a quadratic, we consider a cubic function for growth rate. Then the actual fitted function would be its antiderivative, a quartic polynomial. Therefore, we choose the fit for world population to be

ln(World Population) = Quartic Function of Time, in Years

Hence, the fitted world population function is given by

Fitted World Population = $e^{(\text{Quartic Function of Time, in years})}$

Such thinking yields similar models for the Evangelized and Christian Populations. From here, we proceed in the modeling as in the book.